512. MATHEMATICS

1. **Abstract Algebra :** Automorphisms, Conjugacy and G – sets, Normal series, Solvable groups, Nilpotent groups, Direct product - Finitely generated abelian groups - Invariants of a finite abelian group, Sylow's theorems, Groups of orders p^2 and pq, Ideals and homomorphisms , Sum and direct sum of ideals, Maximal and prime ideals, Nilpotent and nil ideals, Zorn's lemma, Unique factorization domains, Principal ideal domains, Euclidean domains, Polynomial rings over Unique Factorization Domains, Rings of Fractions.

2. **Fields and Galois Theory :** Irreducible polynomials and Eisenstein criterion, Adjunction of roots, Algebraic extensions, Algebraically closed fields, Splitting fields, Normal extensions, Multiple roots, Finite fields, Separable extensions, Automorphism groups and fixed fields, Fundamental theorem of Galois theory, Fundamental theorem of Algebra, Roots of unity and cyclotomic polynomials, Cyclic extensions, Polynomials solvable by radicals, Ruler and Compass constructions.

3. **Mathematical Analysis** : Metric spaces, Compact sets, Perfect sets, Connected sets, Limits of functions, Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities, Monotone functions, Riemann - Steiltjes integral, Definition and Existence of the Integral, Properties of the integral, Integration of vector valued functions - Rectifiable curves, Sequences and series of functions: Uniform convergence Uniform convergence and continuity - Uniform convergence and integration, Uniform convergence and differentiation - Approximation of a continuous function by a sequence of polynomials.

4. **Lebesgue Measure & Integration :** Algebra of sets, Borel sets, Outer measure, Measurable sets and Lebesgue measure, A non – measurable set, Measurable functions, Littlewood three principles, the Riemann integral - The Lebesgue integral of a bounded function over a set of finite measure, The integral of a non - negative function, The general Lebesgue integral, Convergence in measure, Differentiation of a monotone functions, Functions of bounded variation, Differentiation of an integral, Absolute continuity, The L_p – spaces, The Minkowski and Holders inequalities, Convergence and completeness.

5. **Complex Analysis :** Regions in the Complex Plane, Functions of a Complex Variable, Mappings, Mappings by the Exponential Function, Limits, Limits Involving the Point at Infinity, Continuity, Derivatives, Cauchy Riemann Equations, Sufficient Conditions for Differentiability, Analytic Functions, Harmonic Functions, Uniquely determined Analytic Functions, Reflection Principle, The Exponential Function, The Logarithmic Function, Some Identities Involving Logarithms, Complex Exponents, Trigonometric Functions, Hyperbolic Functions, Derivatives of Functions w(t), Definite Integrals of Functions w(t), Contours, Contour Integrals, Some Examples, Examples with Branch Cuts, Upper Bounds for Moduli of Contour Integrals, Anti derivatives, Cauchy Goursat Theorem, Simply Connected Domains, Multiply Connected Domains, Cauchy Integral Formula, An Extension of the Cauchy Integral Formula, Liouville's Theorem and the Fundamental Theorem of Algebra, Maximum Modulus Principle, Convergence of Sequences, Convergence of Series, Taylor Series, Laurent Series, Absolute and Uniform Convergence of Power Series, Continuity of Sums of Power Series, Integration and Differentiation of Power Series, Uniqueness of Series Representations, Isolated Singular Points, Residues, Cauchy's Residue Theorem, Residue at Infinity, The Three Types of Isolated Singular Points, Residues at Poles, Examples, Zeros of Analytic Functions, Zeros and Poles, Behavior of Functions Near Isolated Singular Points, Evaluation of Improper Integrals, Improper Integrals from Fourier Analysis, Jordan's Lemma, Indented Paths, Definite Integrals Involving Sines and Cosines, Argument Principle, Rouche's Theorem, Linear Transformations, The Transformation $w = \frac{1}{z}$, Mappings by $\frac{1}{z}$, Linear Fractional Transformations, An Implicit Form, Mappings of the Upper Half Plane.

6. **Ordinary and Partial Differential Equations :** Existence and Uniqueness of solution of $\frac{dy}{dx} = f(x, y)$ and problems there on, The method of successive approximations, Picard's theorem , Non-Linear partial differential equations of order one, Charpit's method, Cauchy's method of Characteristics for solving non-linear partial differential equations, Linear Partial Differential Equations with constant coefficients. Partial Differential Equations of order two with variable

coefficients - Canonical form , Classification of second order Partial Differential Equations, separation of variables method of solving the one– dimensional Heat equation, Wave equation and Laplace equation, Sturm-Liouville's boundary value problem, Power Series solution of ordinary differential equations, ordinary and Singular points, Series solution about an ordinary point, Series solution about Singular point, Frobenius Method.

Legendre Polynomials : Legendre's equation and its solution, Legendre Polynomial and its properties, Generating function, Orthogonal properties, Recurrance relations , Laplace's definite integrals for $P_n(x)$, Rodrigue's formula.

Bessels Functions : Bessel's equation and its solution, Bessel function of the first kind and its properties, Recurrence Relations, Generating function, Orthogonality properties.

Hermite Polynomials : Hermite's equation and its solution, Hermite polynomial and its properties , Generating function, Alternative expressions (Rodrigue's formula), Orthogonality properties, Recurrence Relations.

7. Theory of Ordinary Differential Equations :

Linear differential equations of higher order : Introduction, Higher order equations, A Modelling problem, Linear Independence, Equations with constant coefficients, Equations with variable coefficients, Wronskian, Variation of parameters, Some Standard methods.

Existence and uniqueness of solutions : Introduction, Preliminaries, Successive approximations, Picards theorem, Continuation and dependence on intial conditions, existence of solutions in the large, existence and uniqueness of solutions of systems, fixed point method.

Analysis and methods of non - linear differential equations : Introduction, Existence theorem, Extremal solutions, Upper and Lower solutions, Monotone iterative method and method of quasi linearization, Bihari's inequality, Application of Bihari's inequality.

Oscillation theory for linear Differential Equation of Second order : The adjoint equation, Self-adjoint linear differential equation of second order, Abel's formula, the number of zeros in a finite interval, The sturm separation theorem, the strum comparison theorem, the sturmpicone the Bocher Osgood theorem, A special pair of solution, Oscillation on half axis.

8. Discrete Mathematics :

Mathematical Logic : Propositional logic, Propositional equivalences, Predicates and Quantifiers, Rule of inference, direct proofs, proof by contraposition, proof by contradiction.

Boolean Algebra : Boolean functions and its representation, logic gates, minimizations of circuits by using Boolean identities and K – map.

Basic Structures : Sets representations, Set operations, Functions, Sequences and Summations, Division algorithm, Modular arithmetic, Solving congruences, applications of congruences.

Recursion : Proofs by mathematical induction, recursive definitions, structural induction, generalized induction, recursive algorithms.

Counting : Basic counting principle, inclucombinations, Binomial coefficient and identities, generalized permutations and combinations, Binomial coefficient and identities, generalized permutations and combinations. **Recurrence Relations** : introduction, solving linear recurrence relations, generating functions, principle of inclusion - exclusion, applications of inclusion - exclusion.

Relations : relations and their properties, representing relations, closures of relations, equivalence relations, partial orderings.

Graphs : Graphs definitions, graph terminology, types of graphs, representing graphs, graph isomorphism, connectivity of graphs, Euler and Hamilton paths and circuits, Dijkstras algorithm to find shortest path, planar graphs, Eulers formula and its applications, graph coloring and its applications.

Trees : Trees definitions, properties of trees, applications of trees BST, Haffman Coding, tree traversals, pre-order, in-order, post-order, prefix, infix, postfix notations, spanning tress DFS, BFS, Prims, Kruskals algorithms.

9. Integral Transforms :

Laplace Transforms : Introduction, Existence theorem, Laplace transforms of derivatives and integrals , Shifting theorems, Transform of elementary functions, Inverse Transformations, Convolution theorem , Applications to ordinary and Partial differential equations.

Fourier Transforms : Introduction, Sine and cosine transforms, Inverse Fourier Transforms (Infinite and Finite Transforms), Applications to ordinary and Partial differential equations.

Hankel Transforms : Introduction, Hankel Transform of the derivatives of a function, Application of Hankel Transforms in boundary value problems, The finite Hankel Transofrm.
Mellin Transforms : Introduction, The Mellin inversion theorem, some elementary properties of Mellin Transforms and Mellin Transfroms of derivatives, Mellin Integrals, Convolution Theorem.

10. Integral Equations & Calculus of Variations :

Volterra Integral Equations : Basic concepts, Relationship between Linear differential equations and Volterra Integral equations, Resolvent Kernel of Volterra Integral equation, differentiation of some resolvent kernels, Solution of Integral equation by Resolvent Kernel, The method of successive approximations , Convolution type equations, Solution of Integro-differential equations with the aid of the Laplace Transformation , Volterra integral equation of the first kind, Euler integrals, Abel's problem, Abel's integral equation and its generalizations.
Fredholm Integral Equations : Fredholm integralequations of the second kind , Fundamentals, The Method of Fredholm Determinants, Iterated Kernels constructing the Resolvent Kernel with the aid of Iterated Kernels, Integral equations with Degenerated Kernels, Hammerstein type equation, Characteristic numbers and Eigen functions and its properties, Green's function, Construction of Green's function for ordinary differential equations, Special case of Green's function, Using Green's function in the solution of boundary value problem.
Calculus of Variations : Introduction, The Method of Variations in Problems with fixed Boundaries, Definitions of Functionals, variation and its properties, Euler's' equation, Fundamental Lemma of Calculus of Variation, The problem of minimum surface of revolution, Minimum Energy Problem Brachisto chrone Problem, Variational problems involving several functions, Functional dependent on higher order derivatives, Euler Poisson equation, Functional dependent on the functions of several independent variables-Euler's equations in two dependent variables, variational problems in parametric form, Applications of Calculus of Variation, Hamilton's principle, Lagrange's Equation, Hamilton's equations.

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